Incorporating Risk in A Positive Mathematical Programming Framework: a Dual Approach

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1. BACKGROUND

Importance of accounting for risk in farmer's behaviour analyses

- Risk is an important component of agricultural activities: market, production, personal, financial, institutional risk (Hardaker *et al.*, 1997)
- Rise in price volatility on agricultural markets
- Most empirical studies show that farmers are risk averse

Importance of calibration of mathematical programming models: e.g. Positive Mathematical Programming (PMP)

Theoretical implication New research frontier in farm modelling: incorporation of farm risk in a PMP framework (a few attempts so far)

Empirical implication Search for farmer's risk management tool

2. RESEARCH QUESTION

We developed a new methodological proposal which incorporates farm risk in a farm level PMP model

We applied this model to investigate the potential role of an AES (grassland program) as farm income stabilisation tool

Farm non linear cost function

$$\max EU(\tilde{\pi}) = E(\tilde{\mathbf{p}}')\mathbf{x} - \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} - \mathbf{u}'\mathbf{x} - \frac{1}{2}\alpha\mathbf{x}'\mathbf{V}\mathbf{x}$$

s.to $\mathbf{A}\mathbf{x} \leq \mathbf{b} \qquad (\mathbf{y})$

$\mathbf{x} \geq 0$

Farm risk component

3. METHODOLOGY – PMP and risk modelling: previous attempts

- Paris & Arfini (2000)
  - mean-variance approach (CARA)
  - 3 step PMP
  - exogenous risk aversion coefficient
- Severini & Cortignani (2011)
  - mean-variance approach (CARA)
  - extension of Heckelei & Wolff
  - simultaneous estimation of non linear cost function, absolute risk aversion coefficient and resource shadow price
- Petsakos & Rozakis (2011)
  - Second order Taylor series expansion
  - logarithmic utility function (DARA)
  - 3 step PMP
  - no estimation of a non linear cost function but ‘rectification’ of a variance matrix

3. METHODOLOGY – PMP and risk modelling: a new proposal

I STEP - estimation

- it merges the I and the II phase PMP by using the dual relationships of a farmer's expected utility maximisation problem
- risk included according to Mean-Variance approach; CARA preferences
- simultaneous estimation of non linear cost term, farmer specific absolute risk aversion coefficients and shadow prices

$$\min \frac{1}{2}\mathbf{u}'\mathbf{u} + \mathbf{y}'\mathbf{b} + \boldsymbol{\lambda}(\bar{\mathbf{x}} + \boldsymbol{\varepsilon}) + \alpha\bar{\mathbf{x}}'\mathbf{V}\bar{\mathbf{x}} + \mathbf{c}'\bar{\mathbf{x}} - E(\tilde{\mathbf{p}}')$$

s.to

$$\mathbf{y}\mathbf{A} + \mathbf{c} + \boldsymbol{\lambda} + \alpha\mathbf{V}\bar{\mathbf{x}} \geq E(\tilde{\mathbf{p}}')$$

$$\mathbf{c} + \boldsymbol{\lambda} = \mathbf{Q}\bar{\mathbf{x}} + \mathbf{u}$$

$$\mathbf{y}, \boldsymbol{\lambda}, \alpha \geq 0$$

Dual economic equilibrium constraint

Objective function – minimises farmer's deviations from non linear cost function and the difference between primal and dual objective function

Constraint linking marginal cost of the I phase with the marginal cost of non linear cost function to be estimated

II STEP - simulation

the calibrated model is used in simulation analyses

$$\max EU(\tilde{\pi}) = E(\tilde{\mathbf{p}}')\mathbf{x} - \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} - \mathbf{u}'\mathbf{x} - \frac{1}{2}\alpha\mathbf{x}'\mathbf{V}\mathbf{x}$$

s. to $\quad \mathbf{A}\mathbf{x} \leq \mathbf{b}$

$$\mathbf{x} \geq 0$$

Farm non linear cost function

Farm risk component

4. EMPIRICAL MODEL & DATA

Farms	3 representative crop farm samples in flat area of Emilia Romagna region differentiated by the farm size (small, 0-20 hectares, medium, 30-100 hectares, large, > 100 hectares)
Crops	sugar beet, common wheat, corn, barley, grassland
Resource constraints	Land
Non linear cost function	Quadratic
Risk	Price risk, Variance-Covariance matrix common to all farms
Data source	AGREA, RICA, Chamber of Commerce of Bologna

4. EMPIRICAL MODEL & DATA

1. Estimation step

Simultaneous estimation of shadow prices, non linear cost function, farmer's specific risk aversion coefficient

Check the model's ability to calibrate to the base year activity levels

2. Simulation step

Simulation scenarios of different levels of crop price volatility

Scenario 1	Scenario 2	Scenario 3	Scenario 4
0.5 baseline volatility	0.1 lower baseline volatility	0.1 higher baseline volatility	1.5 baseline volatility

Check the model's ability to represent farmer's reaction to changes in economic variables

Check the potential role of the grassland program as farmer's income stabilisation tool

Estimation and simulation phases have been applied to each farm sample separately

4. RESULTS – ESTIMATION

- Calibration ability of the model (% deviation between observed and reproduced activity levels) → most small farms deviations < 0.3%
  all medium farms deviations <0.95%
  most large farms deviations < 0.05%

- Farm non linear cost function (**Q** matrix) → non-zero off-diagonal terms of the quadratic matrix and substitution relationships between crops

- Absolute risk aversion coefficients → 5 small farms, 2 medium farms and 5 large farms show risk neutral attitude

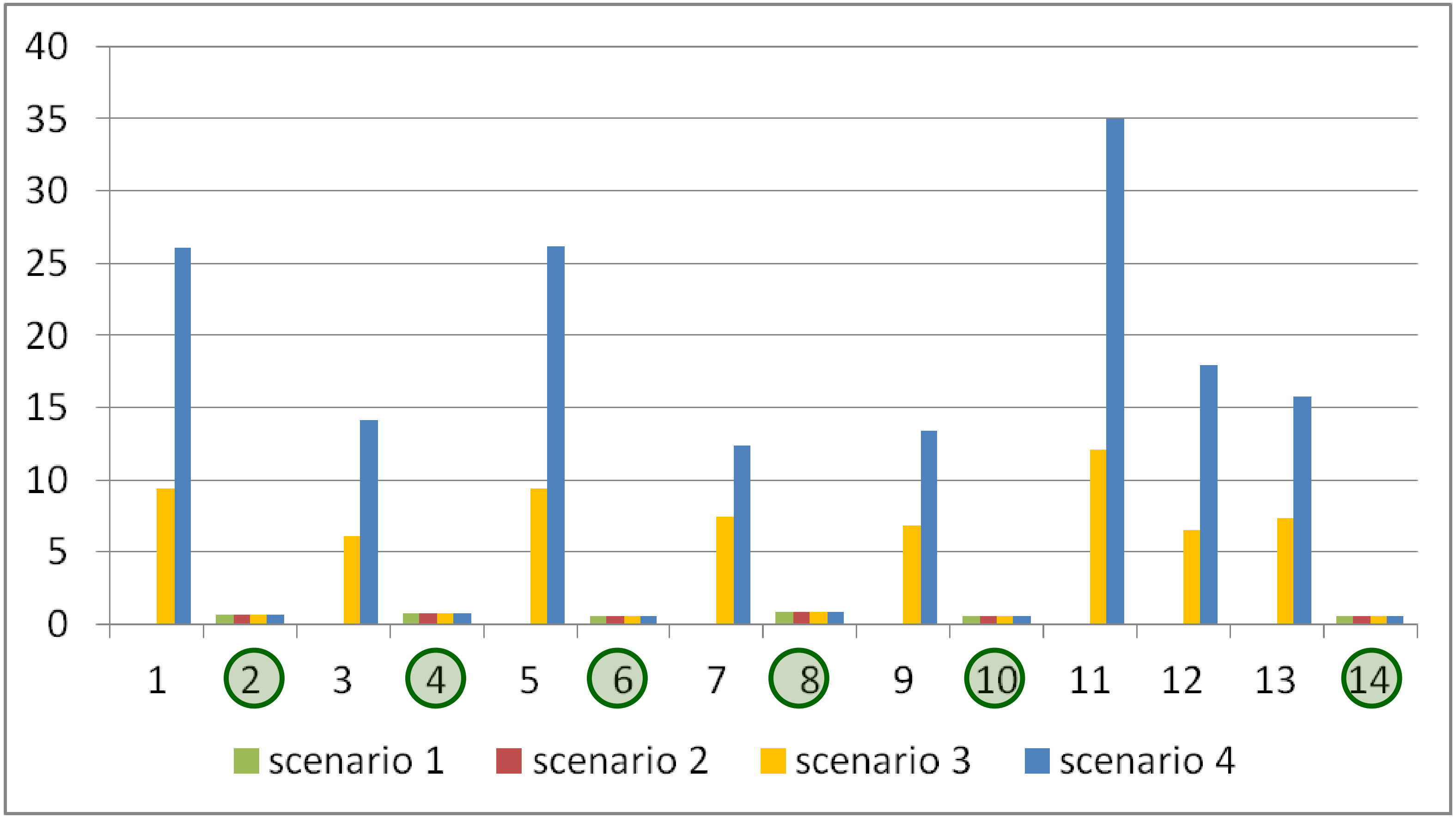
- Relative risk aversion coefficient → Values consistent with the range indicated in the literature 0-7.5 (Chavas & Holt, 1996)

Farmer's relative risk aversion coefficients

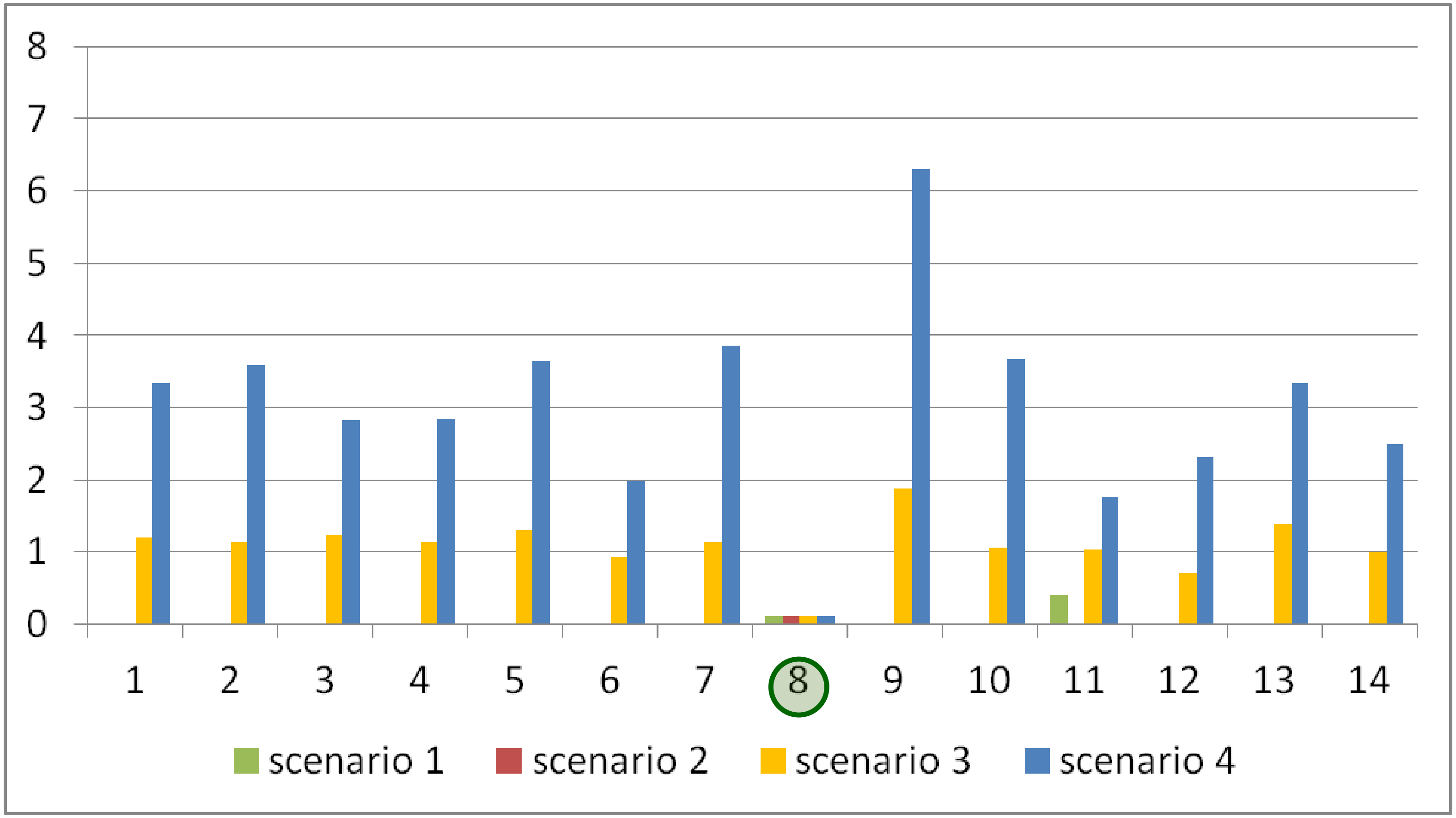
	Small farms	Medium farms	Large farms
1	5.241	11.356	8.105
2	1.728	1.185	1.357
3	3.847	9.024	2.960
4	0.000	0.000	0.000
5	5.738	0.951	0.000
6	0.000	3.992	0.000
7	5.038	4.212	4.085
8	0.000	0.000	0.000
9	6.379	13.355	8.211
10	0.000	2.087	0.635
11	3.422	6.318	3.745
12	4.106	1.268	2.101
13	3.009	2.057	0.428
14	0.000	3.350	0.000

4. RESULTS – SIMULATIONS

Share of farmland committed to AES under different price volatility scenarios in small farm sample

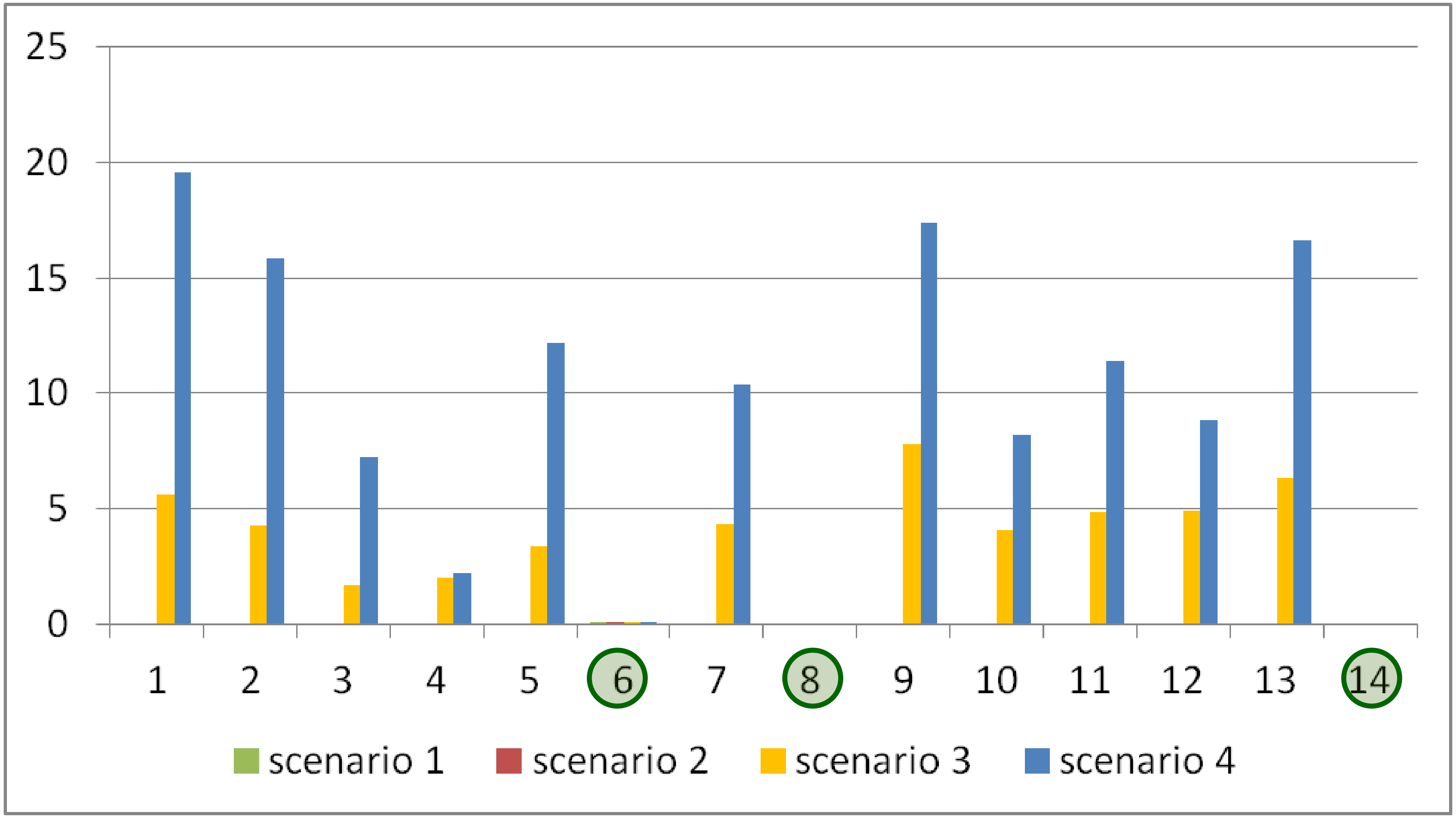


Share of farmland committed to AES under different price volatility scenarios in medium farm sample



4. RESULTS – SIMULATIONS

Share of farmland committed to AES under different price volatility scenarios in large farm sample



5. CONCLUSIONS

- Incorporation of risk in a PMP framework as a new research frontier in farmer's behaviour analyses

- We proposed an innovative methodological approach to incorporate risk in a farm level PMP model which merges the first two phases of the standard PMP and it estimates simultaneously a farm non linear cost function, resource shadow prices and farmer's specific absolute risk aversion coefficient

- The application of the model to three farm samples shows the ability of the model to calibrate to the base year observed activity levels

- The values of the risk aversion coefficients estimated by the model are consistent with the range indicated in the literature (0-7.5)

- Simulation scenarios show the model ability to represent smooth reactions of farmers to changes in economic parameters

- Risk averse farmers increase the share of farmland committed to grassland program when the crop price volatility increases

THANK YOU FOR YOUR ATTENTION

3. METHODOLOGY – standard PMP

Standard PMP (Howitt, 1995): 3 step procedure which uses the dual information of calibration constraints to recover/estimate a farm non linear cost function which calibrates the model to the observed situation

I STEP

$$\max \pi = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x}$$

$$\text{s.to} \quad \mathbf{Ax} \le \mathbf{b} \quad (\mathbf{y})$$

$$\boxed{\mathbf{x} \le \bar{\mathbf{x}} + \boldsymbol{\varepsilon}} \quad (\boldsymbol{\lambda})$$

$$\mathbf{x} \ge 0$$

II STEP

$$\mathbf{c} + \boldsymbol{\lambda} = \mathbf{Qx} + \mathbf{u}$$

III STEP

$$\max \pi = \mathbf{p}'\mathbf{x} - \frac{1}{2}\mathbf{x}'\mathbf{Qx} - \mathbf{u}'\mathbf{x}$$

$$\text{s.to} \quad \mathbf{Ax} \le \mathbf{b} \quad (\mathbf{y})$$

$$\mathbf{x} \ge 0$$

Under-determination problem in the II step

- ad hoc restriction (de Frahan, 2007)
- exogenous supply elasticity (Helming, 2005)
- Generalised Maximum Entropy (Paris & Howitt, 1998; Heckelei & Britz, 2000)

3. METHODOLOGY – PMP and risk modelling: a new proposal

Strengths of our model

- no calibration constraint explicit; it is implicit in the setup of the problem
- simultaneous estimation of shadow prices and non linear cost function avoids the critiques raised against the inconsistency of the shadow prices between step I and step III of the standard PMP

Severini & Cortignani (2011)

- skip the first step of PMP and estimate directly the first order condition of the desired model (econometric perspective)
- farm deviations from optimum activity levels
- do not use information on specific accounting cost per unit of activity

Our proposal

- merge phase I and phase II of PMP by using dual relationships (mathematical programming perspective)
- farm deviations from common non linear cost function
- Use information on specific accounting cost per unit of activity